Algebra 2 *with* TI-nspire

Semester 2

Brendan Kelly

Table of Contents

Unit 7 Rational & Radical Functions & Equations

Unit 8 Exponential & Logarithmic Functions & Equations

Table of Contents

Unit 9 Trigonometric Functions & Equations

Unit 10 Trigonometric Identities

Table of Contents

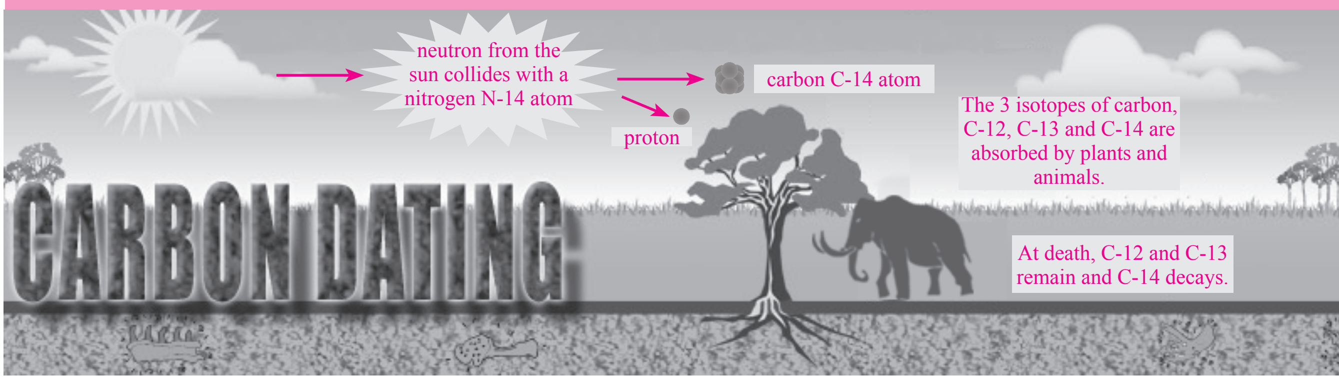
Unit 11 Analytic Geometry: Conics & Polar Coordinates

Unit 12 Permutations, Combinations & Probability

Table of Contents

Unit 13 Statistics

Exploration 48: Exponential Decay



Before 1949, it was difficult to estimate the ages of bones from pre-historic mastodons or wood from ancient Egyptian tombs. Then, in 1949 Willard Libby and his colleagues at the University of Chicago came upon a brilliant idea. They observed that all living organisms contain carbon in three isotopic forms, i.e., C-12, C-13 and C-14. However, when a plant or animal dies, the C-14 isotope decays exponentially while the other isotopes remain constant. By measuring the proportion of C-14 remaining, it is relatively easy to calculate from the rate of decay of the C-14 the number of years ago that the plant or animal has died. This technique is valid for estimating times of death up to 50,000 years ago and was applied by Professor Kullman of Sweden in 2009 to determine the age of the world's oldest tree at more than 8000 years. In 1960, Professor Libby was awarded the Nobel Prize in Chemistry for his discovery of the carbon-dating technique.

Willard Libby

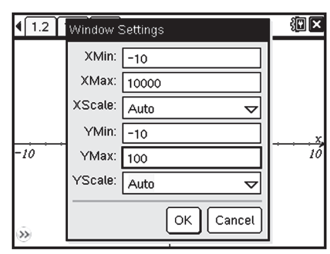
Leif Kullman

Example 1

The percentage y of C-14 in a plant or animal x years after it dies is given by $y = 10^{2 - 0.00005254x}$. Graph y as a function of x and determine the number of years required for the C-14 to decay to 50% of its original mass.

Solution

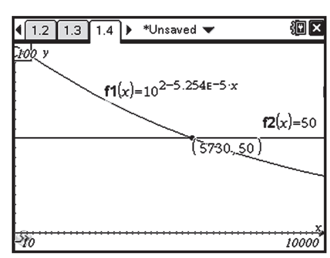
We access the *Graphs* application by pressing (home on) ▸ (enter).
Then we define the function $f1(x) = 10^{2 - 0.00005254x}$ in the entry line. To define an appropriate window for the graph, we press (menu) > **Window /Zoom** > (enter) and complete the template as in the display and click OK.



The C-14 is 50% of its original mass when $y = 50$, so we define $f2(x) = 50$.
To find the point of intersection of $f1(x)$ and $f2(x)$, we press:

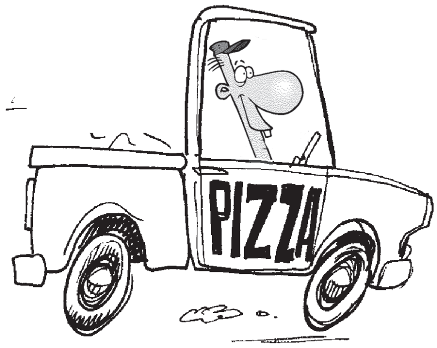
(menu) > **Points & Lines** > **Intersection Point(s)**

Then we click on each graph to obtain the point of intersection (5730, 50). This indicates that the C-14 decays to half its original mass in 5730 years.



Definition: The *half-life* of an exponential function, defined by the equation $f(x) = Ab^x$, is the value of h such that $f(x + h) = ½•f(x)$ for all x.

HOT FROM THE OVEN



In the 1990s, a large pizza company guaranteed delivery in 30 minutes or they would discount the cost. When one of their pizza delivery cars was involved in a car accident, the company was sued for encouraging reckless driving. Why the rush? –because of Newton's Law of Cooling. Most people like their pizza hot, but when it comes out of the oven it begins to cool exponentially!

20 MINUTES LATER

Example 2

The temperature in degrees Fahrenheit of a pizza x minutes after it comes out of the oven and is placed in an insulated delivery bag is given by $f(x) = 75 + 275 \times 10^{-0.028x}$.

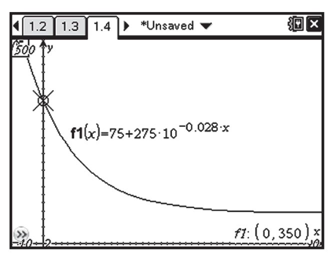
a) Graph $f(x)$ in the window: $-10 \le x \le 90$; $-10 \le y \le 500$.
What is the temperature of the pizza when:
i) it comes out of the oven? ii) it's delivered 20 minutes later?
iii) it's delivered 30 minutes later? iv) it's delivered 1 hour later?
Construct a function table for $f(x)$ to verify your answers.

b) Compare the temperature drop in the first 30 minutes with the temperature drop in the second 30 minutes. Does it make much sense to rush after 60 minutes?

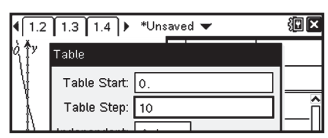
c) How long does it take for the pizza to cool to 76˚F?

Solution

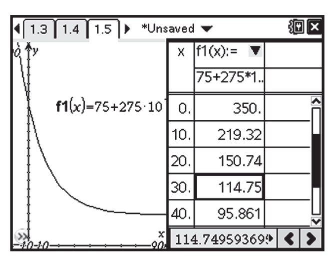
a) We access the *Graphs* application by pressing (on) ▸ (enter).
Then we define the function $f1(x) = 75 + 275 \times 10^{-0.028x}$ in the entry line.
To define an appropriate window, we press (menu) > **Window /Zoom** > (enter) and set the window variables to: $-10 \le x \le 90$; $-10 \le y \le 500$.



To trace along the graph, we press (menu) > **Trace** > **Graph Trace**. Then we press: 0 (enter), 20 (enter), 30 (enter), and 60 (enter) to obtain temperatures of 350˚F, 150.74˚F, 114.75˚F and 80.75˚F respectively.

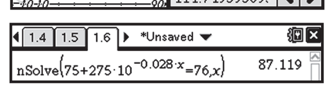


To display a function table for $f1(x)$, we press (menu) > **View** > **Show Table**. To set the table step to 10 minute intervals, we press (menu) > **Table** > **Edit Table Settings** and complete the template as in the display. On clicking OK, we scroll down to verify our answers.



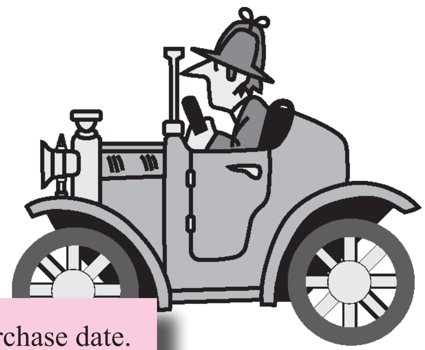
b) In the first 30 minutes the temperature of the pizza falls from 350˚F to 114.75˚F or 235.25˚F, while in the next 30 minutes it falls from 114.75˚F to 80.75˚F or 34˚F. After 60 minutes, the pizza is a few degrees above room temperature, so it's no longer hot.

c) To determine the time it takes the pizza to reach 76˚F, we can find the intersection of the line $f2(x) = 76$ with $f1(x)$ or, alternatively, use the *nSolve* command as in the display. This yields $x = 87.119$ minutes, i.e., it takes about 87 minutes for the pizza to cool to 76˚F.



Worked Examples

Is a car an Investment or an Expenditure?



When you purchase a new car, its value begins to decline from the moment that you drive it off the lot. This gradual decline in value is called *depreciation*. Eventually, the depreciation renders the car worthless. Since an investment is expected to increase in value over time, a car is usually an expenditure.

Example 3

The table shows the value of a luxury car at the end of each year after the purchase date.

The Value of a Car in the First 5 Years after its Purchase						
Years after Purchase	**0**	**1**	**2**	**3**	**4**	**5**
Value in Dollars	46,875	37,500	30,000	24,000	19,200	15,360

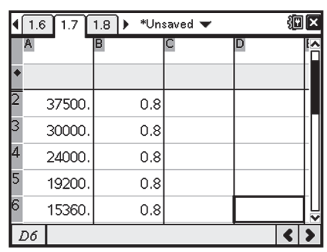
a) Prove that the value of the car depreciates exponentially. What percent of its value does the car lose each year?

b) Derive the equation that expresses the value $f(x)$ of the car x years after its purchase. Graph $f(x)$ as a function of x.

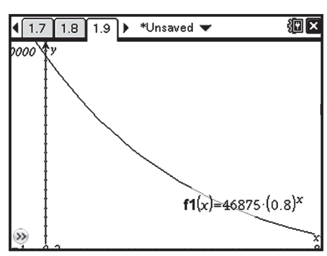
c) Calculate the half-life of the car's value and the year when the car has depreciated to one-quarter of its original value.

Solution

The depreciation is exponential if (and only if) the values of $f(x)$ at any equally-spaced set of values of x form a geometric series, i.e., the ratios of successive terms are constant. Proceeding as in *Example 1b* of *Exploration 47,* we access the *Lists & Spreadsheet* application by pressing (home on) ▶ ▶ ▶ (enter).

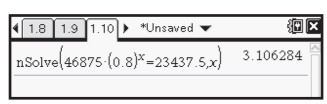


a) We enter the car values in Column A. To find the ratio of the value at the end of each year to the value at the beginning of that year, we enter =A 2 ÷ A 1 in cell *b2*. Then we copy this formula into cells ***b3*** through ***b6***. The number 0.8 appears in cells ***b2*** through ***b6***, indicating that values of $f(n)$ form a geometric series with common ratio 0.8. That is, the value of the car depreciates each year to 0.8 or 80% of its value at the beginning of that year. The car loses 20% of its remaining value each year.



b) It follows from *part a* that $f(x)$ is an exponential function, so $f(x) = Ab^x$ for some constants, A and b.
Since $f(0) = 46{,}875$, $A = 46{,}875$. Since $f(1) = 37{,}500$, then $46{,}875b = 37{,}500$. Therefore, $b = 37{,}500 \div 46{,}875$ or 0.8. Hence $f(x) = 46{,}875 \bullet 0.8^x$.

When we graph $f(x)$ in the window $-1 \le x \le 8$; $-5000 \le y \le 50000$, we obtain the graph in the display, verifying that the car's value decays exponentially.



c) To find the half-life, we could find the intersection of the graph of $f(x)$ with the graph of $y = 46{,}875 \div 2$ i.e., $y = 23{,}437.50$. Alternatively, we use the *nSolve*(command to solve $f(x) = 23{,}437.50$, as in the display. The half-life of the car's value is about 3.106 years. After 3.106 years, the car has decreased to half its original value, and after another 3.106 years, i.e., after 6.2 years, it decreases to one quarter of its original value.

Exercises and Investigations

❶. a) Explain, in your own words the meaning of the term *half-life* as it applies to radioactive decay.

b) Plutonium 238 has a half-life of 88 years. How long will it take for a quantity of plutonium 238 to decay to:
 i) 25% of its original mass? ii) 1/8 of its original mass?

❷. Carbon-14 has a half-life of 5730 years. How much of the original amount of carbon-14 will remain in a plant that has been dead for three times that long, i.e., for 17,190 years?

❸. Tax rules assume that a car loses in a year 30% of the value it had at the beginning of that year.

a) Write an expression for the value of a car after n years (where $n < 10$) if its original cost was $25,000.

b) Display a table showing the value of the car at the end of each of the first 7 years after its purchase.

c) What is the value of the car at the end of the 7th year?

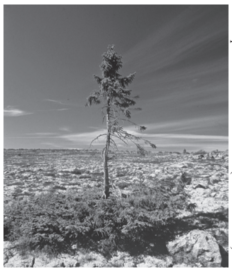
❹. The percentage y of C-14 in a plant or animal x years after it dies is given by $y = 10^{2 - 0.00005254x}$.

a) Graph y as a function of x.

b) Trace along your graph to find the percentage of C-14 that remains after 2000 years.

c) After how many years will only 20% of the original C-14 remain?

❺. In 2008, Professor Kullman discovered the oldest living tree. It is a Norway spruce with a dead branch containing only 38% of its original C-14. Use the formula in *Exercise* ❹ to calculate the age of the tree.



Scanpix/The Canadian Press (Leif Kullman)

❻. Graph, in the window: $-10 \le x \le 90$; $-10 \le y \le 500$, the pizza cooling function $f(x) = 75 + 275 \times 10^{-0.028x}$, from *Example 2*.

a) Trace along your graph to determine the time for the pizza to cool to half its original temperature, i.e., to 175˚F.

b) Use the *nSolve* command to verify your answer in *part a*.

c) Is the time you found in *part a* the half-life of the temperature of the pizza? Explain why or why not.

❼. If the purchasing power of a dollar at the end of every year is 96% of its value at the beginning of the year, how many years does it take to lose half of its value?

❽. The temperature T in degrees celsius of the water in an electric kettle is given by $T = 20 + 80(10)^{-0.04t}$, t minutes after it is brought to a boil and unplugged.

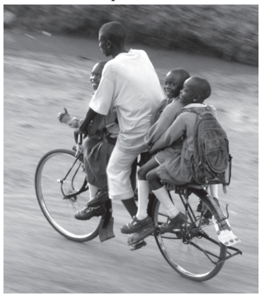
a) Graph the temperature of the water for $0 \le t \le 30$.

b) In how many minutes does the water cool to 50˚C?

c) In how many minutes does the water cool to 30˚C?

❾. If l_m and l_n are the luminosities of stars of magnitude m and n, respectively, then $l_n/l_m = 10^{0.4(m-n)}$. In June 1918, Nova Aquilae increased in luminosity by a factor of 45,000. By how many magnitudes did it rise?

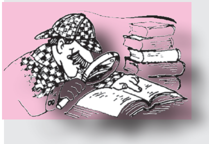
photo by Alex Matheson
www.footprintstours.com



❿. Kenya's population is growing at a faster rate than the populations of most other nations. In 2003, the population of Kenya was estimated to be about 32,000,000. Its annual growth rate was 2.6%. In what year will the population of Kenya reach 50,000,000 if it continues to grow at this rate?

⓫. Suppose $f(t)$ is any exponential function of time t defined by $f(t) = Ab^{-kt}$ where A is a constant, $k > 0$. Prove that if h is the *half-life*, i.e., $f(h) = ½ f(0)$, then $f(t + h) = ½ f(t)$ for all values of t.

TI-nspire Investigation



Discovering $\log_{10} x$

Open the file *logfunction.tns* or follow these instructions.

In the entry line of the *Graphs* application, press these keys [ctrl] [10^x] [1] [0] [tab] [X] [enter] to define $f1(x) = \log_{10} x$.

a) To evaluate $\log_{10} 1$, enter [menu] > **Trace** > [enter] [1] [enter].

b) Press [menu] > **Trace** > [enter] [1] [0] [enter] to evaluate $\log_{10} 10$.

c) Press [menu] > **Window /Zoom** > [enter] and set the window variables to: $-10 \le x \le 1000$; $-6 \le y \le 6$. Trace to evaluate $\log_{10} 100$ and $\log_{10} 1000$.

d) Conjecture an expression for $\log_{10}(10^n)$ where n is a positive integer. Use this expression to conjecture the values of $\log_{10} 10{,}000$ and $\log_{10} 100{,}000$.

e) In the *Calculator* application, evaluate $\log_{10} 10{,}000$ and $\log_{10} 100{,}000$ to check your conjecture in *part d*.

f) Graph the function $f(x) = \log_{10}(10^x)$. Conjecture an identity relating x and $f(x)$.

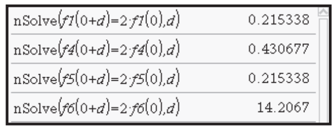
g) Graph the function $g(x) = 10^{\log_{10} x}$.
Conjecture an identity relating x and $g(x)$.

h) Use your conjectures to evaluate $\log_{10} 10^{1.017}$ and $10^{\log_{10} 1.017}$.

i) Describe a relationship between the functions $y = 10^x$ and $y = \log_{10} x$.

Exploration 47 *cont'd*

❽. The display verifies the doubling time for each of the functions in *Exercise* ❼. Since we have established that these are exponential functions, we know that the change in $f(x)$ is the same on any interval of length d. Therefore, we can evaluate $f(x)$ at $x = 0$ and $x = d$ to find the doubling time.



❾. The population growth in the period from 1960 to 1990 is given by $f(1990) - f(1960)$ where $f(x) = 0.000014(1.017)^x$. Substituting 1990 and 1960 for x yields a population growth of about 2.058 billion in 30 years. This is an average growth rate of 2.058 ÷ 30 or 68.61 million per year. The population growth from 1990 to 2009 was 6.8 billion – $f(1990)$ or about 1.6 billion, in 19 years. The average growth rate from 1990 to 2009 was 1.6E9 ÷ 19 or about 84.95 million per year. The rate of population growth from 1990 to 2009 was greater then from 1960 to 1990.

❿. a) If d is the doubling time for the function $f(x) = Ab^x$, then $Ab^{x+d} = 2Ab^x$. That is, $Ab^xb^d = 2Ab^x$, so $b^d = 2$. ①
Similarly, if δ is the doubling time for $f(x) = b^x$, then $b^{x+\delta} = b^x b^\delta = 2b^x$, so $b^\delta = 2$. ②
Comparing ① and ② yields $d = \delta$, so Ab^x and b^x have the same doubling time.

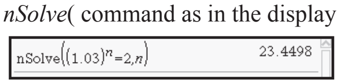
b) The doubling time d for the exponential function $f(x) = b^x$ is the solution of the equation $b^{x+d} = 2b^x$ for d. Multiplying both sides of this equation by b^{-x} yields $b^d = 2$. To solve $b^d = 2$, using the *nSolve*(command, we write *nSolve*($b^d = 2, d$) or, equivalently, *nSolve*($b^x = 2, x$).

⓫. If $f(x) = b^x$ doubles every two years, then $f(x + 2) = 2f(x)$, that is, $b^{x+2} = 2b^x$, i.e., $b^xb^2 = 2b^x$, so $b^2 = 2$. Therefore $b = \sqrt{2}$.
Therefore, $f(x)$ increases by about 141.4% each year.

⓬. The population reached 500,000 on the 15th day.

⓭. The population increases by a factor of 1.12 each year. Therefore, it increases by a factor of 1.12^{10} or about 3.11 at the end of 10 years. That is, it more than triples in 10 years.

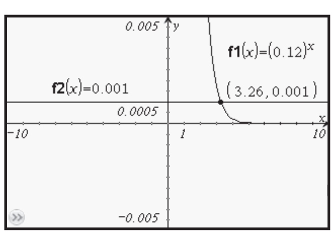
⓮. The investment doubles in value after n years where n satisfies the equation $100000(1.03)^n = 200000$, i.e., $(1.03)^n = 2$.
To find the value of n we enter the *nSolve*(command as in the display to obtain $n \approx 23.45$. That is, the investment doubles in about 23½ years.



⓯. a) Let $f(x) = Ab^x$. Then, $Ab^{1.2} = 15.8341$ ① and $Ab^{3.5} = 384$ ②.
Dividing ② by ① yields $b^{2.3} = 384/15.8341$, from which we find $b = (384/15.8341)^{1/2.3}$ or 4. Substituting into equation ① or ②, we find $A = 3$. Therefore, the function is $f(x) = 3•4^x$.

b) Proceeding as in *part a*, we obtain $b = [f(p)/f(q)]^{1/(p-q)}$ and $A = f(p)^{q/(q-p)}f(q)^{p/(p-q)}$.

⓰. We can write $12^x > 10^{2x-3}$ as the inequality $12^x/10^{2x} > 1/10^3$, which can be written as: $(12/100)^x > 1/10^3$, or $(0.12)^x > 0.001$. By graphing or using *nSolve*(, we find $0.12^x = 0.001$ at $x = 3.26$. Therefore, $12^x > 10^{2x-3}$ for $x < 3.26$.



Exploration 47 *cont'd*

⓱. A function $f(x)$ is exponential if $f(x + 2d)/f(x + d) = f(x+d)/f(x)$ for any value of d. Using this test we find the following results.
a) For $d = 0.5$, $f(x+d)/f(x) = 2$, so this is an exponential function.
b) For $d = 3$, $f(3 + d)/f(3) = 1.06…$, but $f(6 + d)/f(6) = 1.07…$, so this table does not define an exponential function.
c) For $d = 3$, $f(3.4 + d)/f(3.4) = 1.225…$ and $f(6.4 + d)/f(6.4) = 1.225…$ and this ratio is the same for all adjacent x values in the table so this table defines an exponential function.

⓲. Using the method in *Exercise* ⓯, we find the following equations for the exponential functions defined in ⓱a and ⓱c.
a) $f(x) = 4^x$ b) $f(x)$ is not exponential c) $f(x) = 3(1.07)^x$

⓳. To grow 8-fold, it must double 3 times. Since it doubles every 2 years, it will double 3 times or 8-fold in 6 years.

⓴. An exponential function $f(x)$ is characterized by the property that ratios of successive values of $f(x)$ on a sequence of equally-spaced values of x are constant. That is, for any fixed d, the ratios of successive terms of the sequence $f(x)$, $f(x + d)$, $f(x + 2d)$, … is a fixed constant, say k. Then the sequence $f(x)$, $f(x + d)$, $f(x + 2d)$, … can be written as the sequence $f(x)$, $kf(x)$, $k^2f(x)$, $k^3f(x)$ … The successive differences of this sequence are $(k - 1)f(x)$, $(k^2 - k)f(x)$, $(k^3 - k^2)f(x)$, …and the ratios of the successive terms in this sequence are the constant k. That is, the consecutive differences are growing exponentially.

Exploration 48

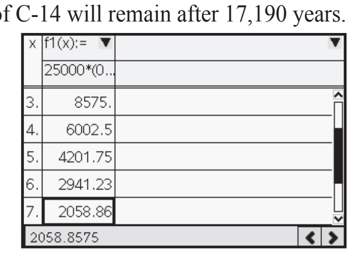
❶. a) The half-life of a mass of radioactive material is the time it takes to decay to half of its present value.
b) i) 176 years ii) 264 years

❷. One-eighth of the original amount of C-14 will remain after 17,190 years.

❸. a) After n years the value of the car is $\$25000(0.7)^n$.

b) The display shows the values of the car each year after purchase.

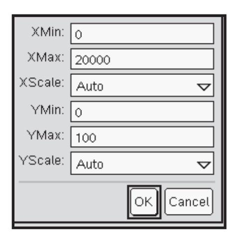
c) The table shows that at the end of the seventh year, the car is worth $2058.86.

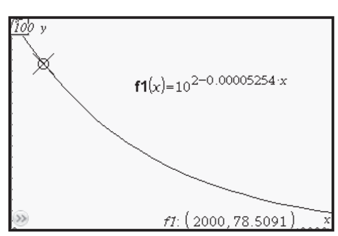


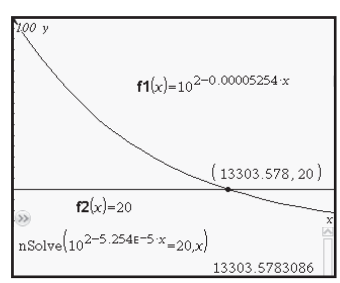
❹. a) The top display shows the window settings and the display, below left, shows the graph.

b) The display below left shows that 78.5% of the original C-14 remains after 2000 years.

c) The display, below right, shows that the graph intersects the line $y = 20$ at the point (13,303, 20). That is, there is 20% of the C-14 remaining after 13,303 years.



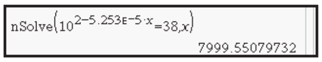




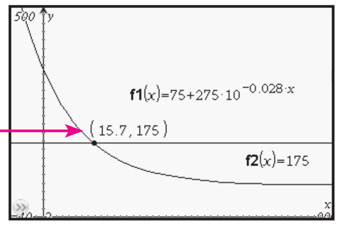
Answers to the Exercises & Hints for the Investigations

Exploration 48 *cont'd*

❺. In *Example 1*, we learned that the percentage of C-14 in a plant x years after its death is given by $10^{2-0.00005253x}$. To find the age of the Norway spruce containing 38% of its original C-14, we access *Calculator* and enter *nSolve*$(10^{2-0.00005253x} = 38, x)$ to obtain $x = 7999.55$ as shown in the display. That is, the Norway spruce is about 8000 years old.



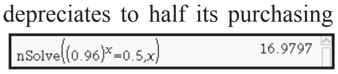
❻. a) We can trace along the graph to the point with y-coordinate 175, or intersect the graph with the line $y = 175$ to find that the pizza cools to 175˚ in 15.7 minutes.



b) The *nSolve*(command in the lower display verifies the answer in *part a*.

c) 15.7 minutes is not the half-life of the temperature of the pizza, because the temperature $f1(x)$ not an exponential function, but the sum of an exponential function and a constant.

❼. If the dollar at the end of each year is 96% of the purchasing it had at the beginning, its value at the end of n years is 0.96^n. To determine the number of years to reach 50% of its value, we must solve $0.96^n = 0.5$. The display shows that the dollar depreciates to half its purchasing power in about 17 years.

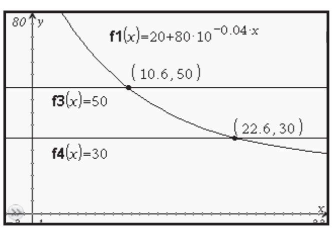


❽. a) The display shows the graph in the window:

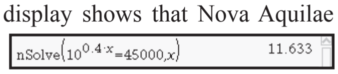
$-3 \le x \le 33; -5 \le y \le 80.$

b) The display shows that the water cools to 50˚C in 10.6 minutes.

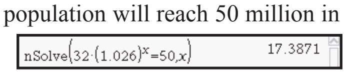
c) The display shows that the water cools to 30˚C in 22.6 minutes.



❾. If x denotes the difference in magnitude of the Nova Aquilae, and l_{new} and l_{old} denote the luminosities after and before the supernova, then $l_{new} \div l_{old} = 45{,}000 = 10^{0.4x}$. The display shows that Nova Aquilae increased by 11.6 magnitudes.



❿. The population of Kenya grows to $32(1.026)^x$ million in x years. The display shows that, at this rate, the population will reach 50 million in about 17 years.



⓫. If h is the half-life, then $f(h) = ½f(0) = ½\,A$. ①

Also $f(h) = Ab^h$. ②

Comparing ① and ②, we obtain $b^h = ½$

$$f(t+h) = Ab^{t+h} = b^h(Ab^t) = b^h f(t) = ½\,f(t).$$

Exploration 49

❶. The functions $f(x) = 10^x$ and $g(x) = \log x$ are inverse functions. Therefore, $f(g(x)) = x$, i.e., $10^{\log x} = x$. Also, $g(f(x)) = x$, i.e., $\log 10^x = x$.

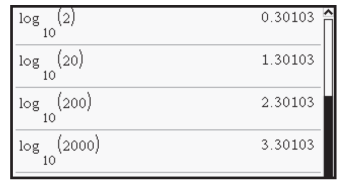
❷. a) 3 b) 5 c) –3

❸. a) Raising both sides to powers of 10 yields: $10^y = x$.

b) If $\log x = 4$, then $10^{\log x} = 10^4$. That is $x = 10{,}000$.

Exploration 49 *cont'd*

❹. The display shows the values of log 2, log 20, log 200, and log 2000 where the base of the log function is assumed to be 10.



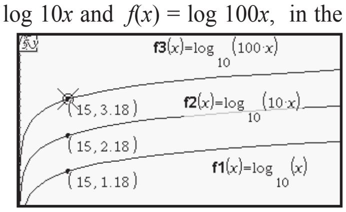
❺. Each answer in *Exercise* ❹ is 1 greater than the previous answers. Multiplying a number by 10 increases its logarithm (base 10) by 1.

❻. a) By definition, $10^{\log 2} = 2$ and $10^{\log 20} = 20$, therefore $10^a = 2$ and $10^b = 20$. That is, $10^b = 10 \times 10^a$. Using the exponent law for multiplying powers of 10, we have $10^b = 10^{a+1}$ so, $b = a + 1$.

b) The derivation in *part a* verifies the observation in *Exercise* ❺ that multiplication by 10 increases the logarithm of a number by 1.

c) The graph of log $10x$ is the graph of log x translated one unit vertically.

❼. a) Graphing $f1(x) = \log x$, $f2(x) = \log 10x$ and $f(x) = \log 100x$, in the window $0 < x \le 100$; $-1 \le y \le 5$, yields the display on the right.



b) Tracing the graph $y = \log x$, to the point (15, 1.18), and pressing the "up cursor" key moves the cursor to the point (15, 2.18) on the graph of $y = \log 100x$. In general, if (x, y) is an arbitrary point on the graph, of log x, then the points on the graphs of log $10x$ and log $100x$ directly above (x, y) are $(x, y + 1)$ and $(x, y + 2)$ respectively.

❽. a) Since $\log x = 2.4$, then $10^{\log x} = 10^{2.4}$. We compute $10^{2.4}$ directly, as 10^2.4 and obtain 251.1886432… That is, $10^{2.4} = 251.1886432…$ Since $10^{\log x} = x$, it follows that $x = 251.188…$

b) We can proceed as in *part a* and compute x = 10^3.8 which is 6309.573… Alternatively, we can use the property discovered in *Exercise* ❼ and reason as follows: from *part a*, log 251.1886432 = 2.4, therefore, log 2.51188…= 0.4. Therefore, 2log2.51188… = 0.8. That is, $\log 2.51188^2 = 0.8$. Squaring yields log 6.3095… = 0.8, and so, from *Exercise* ❼, we conclude that log 6309.5… = 3.8; that is, $x = 6309.5$.

c) In *part b*, we found log 6.3095… = 0.8, so $x = 6.3095$.

d) We can either compute x directly as $10^{-2.4}$, or compute it as the reciprocal of $10^{2.4}$ which we found in *part a* to be 251.188… In either case, we find, $x = 0.00398…$

❾. Taking the logarithm (to base 10) of both sides of the equation, we obtain $\log l_n/l_m = 0.4(m - n)$. Therefore, $m - n = 2.5 \log l_n/l_m$. This can also be written as $m - n = 2.5 (\log l_n - \log l_m)$.

❿. Applying the multiplicative property of logarithms, we write: $\log x + \log x^{-1} = \log x \cdot x^{-1}$ which yields log 1, which is 0. That is, the logarithm of a number x, where $x \ge 0$, and its reciprocal add to zero.

⓫. a) The graphs are shown in the display.

b) The graph of $f3(x)$ is shown.

c) Tracing along $f3(x)$ reveals it is the line $y = 0.699…$

d) Since 0.699 = log 5, *part c* suggests that the graph of log $5x$ is the graph of log x translated log 5 units upward. We conjecture that $y = \log Ax$ is the graph of log x translated vertically upward log A units.