Algebra 1 with TI-nspire

Semester 2

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(home on)

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Just press (scratchpad).

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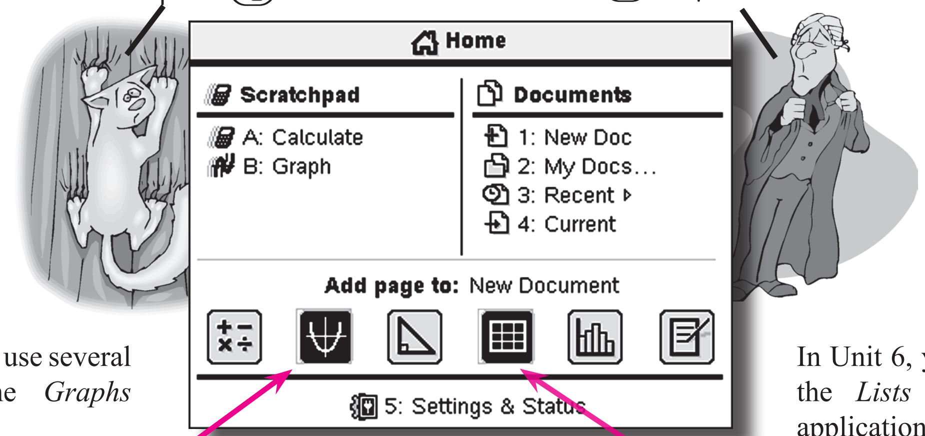

In Unit 6, you will use several submenus in the *Graphs* application.

In Unit 6, you will also use the *Lists & Spreadsheet* application.

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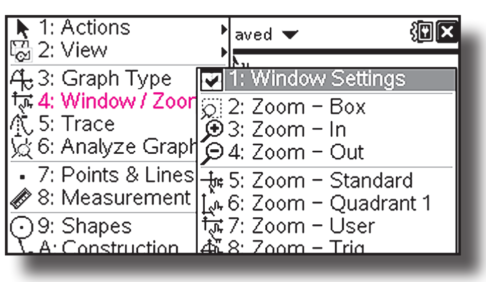
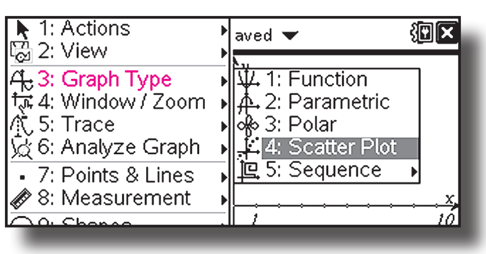
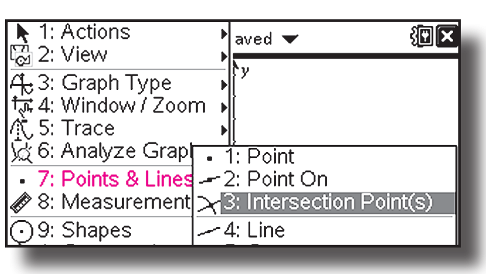
Click on this icon…then press (menu) to obtain the *Lists & Spreadsheet* menu.

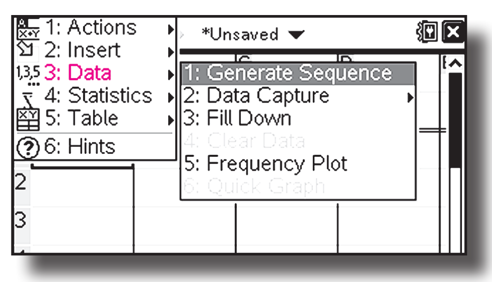
Some Graphs Sub-Menus You Will Need in this Unit

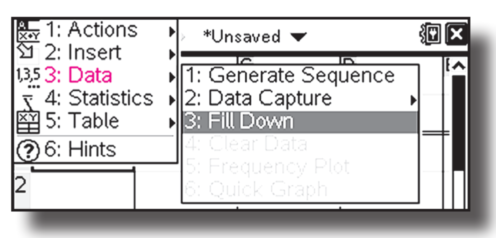


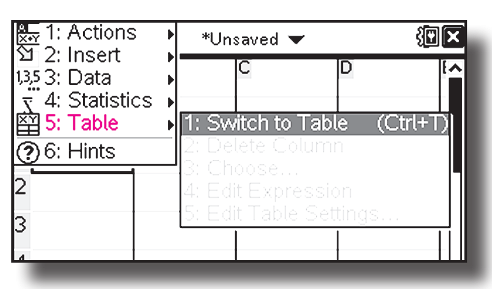




Lists & Spreadsheet Sub-Menus You Will Need in this Unit

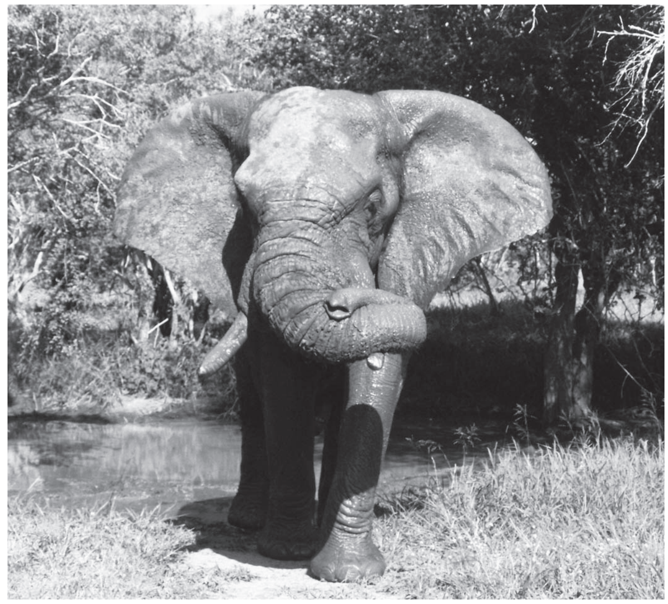






Exploration 52: Data Graphs, Mean, Median & Quartiles

The African elephant is the world's largest land mammal. Males can weigh up to 7 tons and stand 12 feet tall at the shoulder. This magnificent creature characterizes the rich diversity of the exotic African wildlife.



In 1950 there were about 4,000,000 elephants roaming across the African continent. However, this changed dramatically in the decades that followed. Ivory hunters, slaughtering these giants in great numbers, brought the elephant populations to the brink of extinction.

In 1989 CITES, a UN sponsored organization, placed the African elephant on the endangered species list. This classification prohibited international sales of ivory. However, poachers in some areas continued to kill elephants and sell their ivory in illegal markets.

When elephant populations were high, ivory hunters killed only the large bull elephants. As the populations dwindled, poachers began killing females and young elephants. When a large caché of 40 elephant tusks were discovered, scientists were called in to weigh the tusks and determine the ages of the elephants slaughtered.

Example 1

a) Construct a dot plot to show the distribution of the masses of the tusks given in the table.

b) Use your dot plot to determine the number of tusks with a mass of 22 kg.

Masses of the Tusks in kilograms

14	**22**	**16**	**9**	**8**	**13**	**21**	**6**
36	**17**	**20**	**5**	**35**	**36**	**22**	**16**
31	**16**	**23**	**13**	**3**	**12**	**8**	**10**
20	**35**	**14**	**22**	**3**	**36**	**37**	**7**
8	**10**	**31**	**3**	**17**	**11**	**8**	**3**

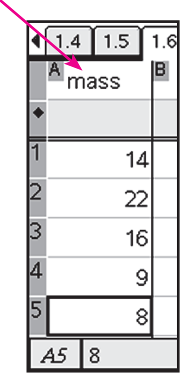
Solution

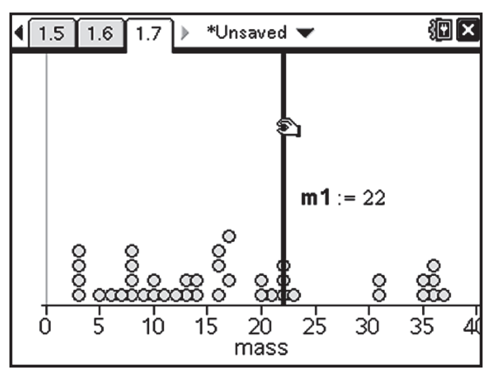
a) To enter these data into a spreadsheet, we press: (on) ▸ ▸ ▸ (enter) and enter into Column A of a spreadsheet the masses given in the table.

Then we type the word *mass* in the cell at the top of Column A (as shown in the display) to name the data.

To access the *Data & Statistics* application, we press: (on) ▸ ▸ ▸ ▸ (enter). We move the cursor to the horizontal axis and click when the prompt "Click to add variable" appears. We choose *mass*, press (enter) and we obtain the dot plot displayed on the right.

b) To insert a movable line, we press: (menu) > **Analyze** > **Add Movable Line**, and drag the line horizontally until it displays 22. The 3 dots on this line indicate that there are 3 tusks with mass 22 kg.





Example 2

a) Display the masses of the tusks in a histogram to show how many tusks have a mass x in each interval:
$0 \le x < 5$ kg, $5 \le x < 10$ kg, … , $35 \le x < 40$ kg.

b) Use your histogram to determine the number of tusks in each interval:
(i) 10 kg $\le x < 15$ kg. (ii) 26 kg $\le x < 30$ kg.

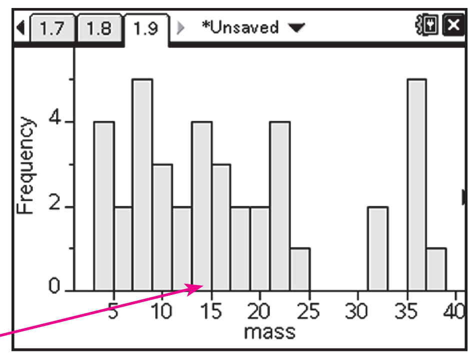
c) What does the shape of your histogram suggest about the ages of the elephants that were killed?

Masses of the Tusks in kilograms

14	22	16	9	8	13	21	6
36	17	20	5	35	36	22	16
31	16	23	13	3	12	8	10
20	35	14	22	3	36	37	7
8	10	31	3	17	11	8	3

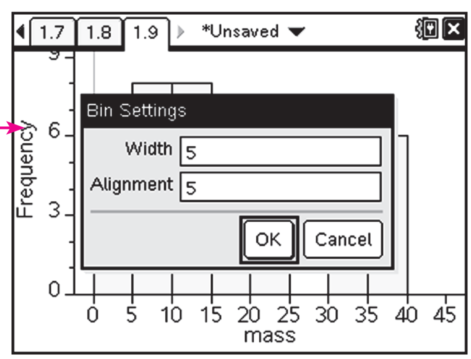
Solution

a) To open a new spreadsheet, using the data already entered, we press: (on) ▸ ▸ ▸ (enter) and at the top of Column A we type the word *mass*. Column A fills with the masses of the tusks as in *Example 1*.



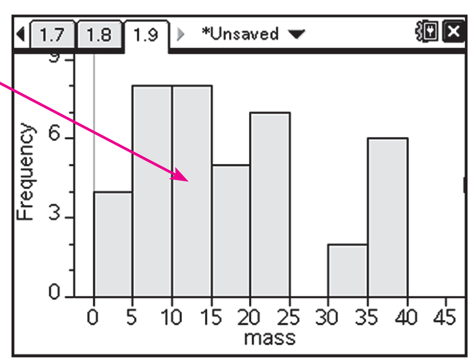
To add a new page with the *Data & Statistics* application, we press: (on) ▸ ▸ ▸ ▸ (enter). We move the cursor to the horizontal axis and click when the prompt "Click to add variable" appears. We choose *mass*, press (enter) and we obtain the dot plot displayed in *Example 1.*

To obtain a histogram, we press (menu) > **Plot Type** > **Histogram** and we obtain the histogram shown in this display. This histogram shows the number of tusks of each particular mass.



To group the masses in 5-kg intervals, we press:
(menu) > **Plot Properties** > **Histogram Properties** > **Bin Settings**
and complete the Bin Settings template as shown and click OK.

If the bars extend beyond the top of the display window, we press:
(menu) > **Window/Zoom** > **Zoom - Data**
We obtain the histogram shown in the bottom display.



b) To determine the number of tusks with mass in a particular interval, we merely observe the height of the bar corresponding to that interval.

(i) Since the bar corresponding to the interval 10 kg $\le x <$ 15 kg has a height of 8 units, we conclude that there are 8 tusks with mass in this range.

(ii) Since the bar corresponding to the interval 25 kg $\le x <$ 30 kg has a height of 0 units, we conclude that there are no tusks with mass in this range. (Note that this interval does not contain the mass 30 kg.)

c) The histogram shows that only 8 of the 40 tusks, i.e., 20% have mass greater than 25 kg. This suggests that there were only about 4 of the large male elephants killed (each contributing 2 tusks) and the other 32 were females and young elephants.

Worked Examples

When the police came upon the campsite of a poacher, they discovered a small caché of tusks. The smaller tusks were clearly taken from a young elephant. The masses of the larger tusks were 12 kg and 13 kg. They compared these with the caché discussed in *Examples 1* and *2* to determine whether these were larger or smaller than the average tusk in the larger caché. To make this comparison, they used a *box-and-whisker* plot (see *Example 3* and *Exercise 4*).

the abandoned campsite of an ivory poacher

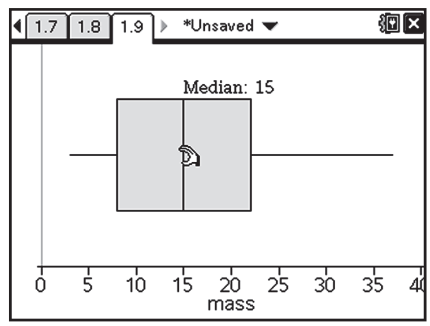
Example 3

Construct a box-and-whisker plot to display the masses of the tusks in *Example 1*. Use the plot to determine:

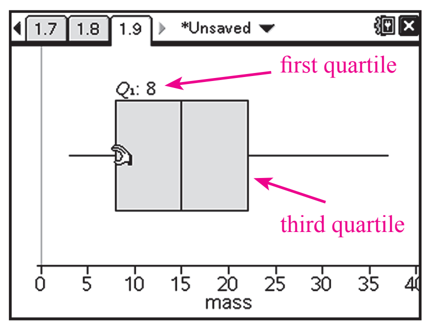
a) the median mass of the elephant tusks.
b) the first and third quartiles of the masses.
c) the masses of the largest and smallest tusks.

Solution

a) To construct a box-and-whisker plot, we construct a dot plot as in *Example 1*. Then, we press (menu) > **Plot Type** > **Box Plot**. We obtain what is called a *box-and-whisker plot* shown in the top display.

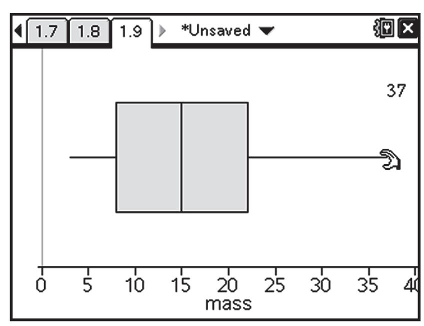


We use the Touchpad to move the pointer (which becomes an open hand icon) toward the middle of the box plot. The screen displays *Median: 15*. This means that half of the tusks have mass that is greater than 15 kg and half have a mass that is less than 15 kg. The *median,* 15, is the middle value.



b) When we move the open hand icon close to the left edge of the box, the screen displays Q_1: 8, indicating that the *first quartile* is 8. That is, the smallest quarter of the tusks have mass that is 8 kg or less.

When we move the open hand icon close to the right edge of the box, the screen displays Q_3: 22, indicating that the *third quartile* is 22. That is, the largest quarter of the tusks have mass that is 22 kg or more.



c) When we move the hand icon to the line segment left of Q_1, (called a *whisker*) the display shows 3, indicating that the smallest tusk has a mass of 3 kg. Moving the hand icon to the whisker right of Q_3, causes a 37 to appear on the screen indicating that the largest tusk has a mass of 37 kg.

We formalize these terms as follows:

Definition:

When a data set is ordered from smallest to largest, the *median* is the middle value if there is an odd number of elements and the mean of the middle values if an even number of elements. The median of the "lower" half of the data set is called the *first quartile* and the median of the "upper" half of the data is called the *third quartile.*

Exercises and Investigations

❶. Explain in your own words the meaning of each measure for a set of data.
a) mean b) median c) first quartile d) third quartile

❷. To find the median and quartiles of a set of data manually, we can proceed as follows.

step 1: list the data in order from smallest to largest.

step 2: If there is an odd number of data, record the middle number as the median. If there is an even number of data, take the mean of the two middle numbers and record that as the median.

step 3: Exclude the median and divide the remaining data into the smaller half and the larger half.

step 4: Find the median of the smaller half and record that as the first quartile, $Q1$.

step 5: Find the median of the larger half and record that as the third quartile, $Q3$.

Use the process above to find the quartiles of each data set.
a) {15, 16, 29, 36, 45, 16, 18, 29, 16, 29, 32, 36}
b) {-17, 61, 42, 156, -88, -61, 42, 42, 61, -17, -17}

❸. Determine the mean and median of each set of numbers.
a) {1, 2, 3, 4, …99} (the natural numbers from 1 to 99)
b) {1, 2, 3, 4, …100} (the natural numbers from 1 to 100)
c) {-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}

❹. a) Enter into Column A of a spreadsheet the 40 masses of the tusks given in *Example 1*. Construct the dot plot as in *Example 1*.

b) Construct the box-and-whisker plot as in *Example 3* and use it to calculate the quartiles of the tusk data.

c) To calculate the mean of the masses of the tusks, press: (doc▾) > **Settings & Status** > **Settings** > **General** and choose *Approximate* for the **Calculation Mode**. Then access your spreadsheet and enter in Column B1 the expression:

=sum(*a1:a40*)/40

Record the mean mass.

d) Which measure, the mean or the median, do you think is a better indicator of the typical mass of the elephant tusks in the caché? Give a reason for your answer.

❺. In *Example 3* we constructed the box-and-whisker plot for the masses of 40 tusks found in a caché. The largest two tusks found in the abandoned campsite had masses of 12 and 13 kg.

a) In what quartile of the 40-tusk caché do these masses fall?

b) If the masses of those two tusks were added to the data set including the 40 masses, would the median change? Explain.

c) Do you think that the 12-kg and 13-kg tusks were from large elephants? Give a reason for your answer.

❻. In a study of the longevity of a particular species of cat, biologists recorded the life spans of 30 cats. Their results are presented in the following table.

Life spans of Cats (in years)

12.9	13.2	14.1	13.9	12.8	13.1	13.1	13.2	13.6	13.0
13.4	13.6	12.9	13.3	11.8	12.8	14.6	12.8	10.4	14.8
11.5	13.5	13.6	12.9	9.6	14.5	13.5	13.8	14.4	13.3

a) Construct a dot plot to display the data in the table. How many cats had a lifespan of 13.6 years?

b) Construct a box-and-whisker (meow) plot to find the first, second and third quartiles in the lifespan data set. Record the shortest and longest life spans.

c) Use your spreadsheet to calculate the mean lifespan of the cats (see *Exercise* ❹c)

d) Construct a histogram showing the number of life spans in the intervals of bin width 0.5 years between 9.5 and 15 years. That is, $9.5 \le x < 10$, $10 \le x < 10.5$, …, $14.5 \le x < 15$. How many life spans in the interval $13.5 \le x < 14.0$?

TI-nspire Investigation

How Many Dimples On a Golf Ball?

A golf ball without dimples would not travel very far. The dimples give it lift that increases the distance it travels. A golf magazine published the number of dimples on 32 different brands of golf ball (shown in the table below).

Number of Dimples on each Golf Ball

440	432	392	432	318	442	432	432
432	360	360	360	432	492	440	392
392	492	332	422	422	422	332	440
392	332	432	432	392	332	392	360

a) Enter these data into Column A of a spreadsheet. Type “Dimples” in the top row of Column A. Press (menu) > **Statistics** > **Stat Calculations** > **One-Variable Statistics** > (enter). Scroll in Column C to find the mean, $\overline{x}$, the median (MedianX), the quartiles (Q_1X and Q_3X), and the smallest and largest values.

b) Construct a box-and-whisker plot to verify your answers in *part a*.

c) Display these data in a histogram with bin size 20 and record the percentage of golf balls with dimples in the interval $420 \le x < 440$.

Answers to the Exercises & Hints for the Investigations

Exploration 52

❶. The mean of a set of n numbers is their sum divided by n. The definitions of the first, second and third quartiles are given in the paragraph following *Example 3*.

❷. The easiest way to apply the procedure described in this exercise to determine the quartiles, is to enter the data into Column A of a spreadsheet, highlight the header row and select the Sort command from the Actions sub-menu of the spreadsheet. This lists the data in ascending or descending order and enables you to identify the middle number in any subset.

a) Q_1: 16, Q_2: 29, Q_3:34. b) Q_1: –17, Q_2: 42, Q_3:61.

❸. a) The sum of the numbers from 1 to 99 is (99 × 100)/2 = 4950
The mean of these numbers is 4950 ÷ 99 = 50.
There are 99 numbers and the middle number is 50, so the median is 50.

b) The sum of the numbers from 1 to 100 is 4950 + 100 = 5050.
The mean of these numbers is 5050 ÷ 100 = 50.5.
There are 100 numbers, so the two middle numbers are 50 and 51. The mean of these two numbers is 50.5, so the mean and median of the set of integers from 1 to 100 are both 50.5

c) The sum of the numbers in this set is –21, so the mean is –21 ÷ 14 or –3/2. The two middle numbers in this set are –2, – 1, so the median is –3/2.

❹. a) See *Example 1* b) See *Example 3*
c) The sum of the masses is 677 kg.
The mean mass is 677 ÷ 40 = 16.925 kg.

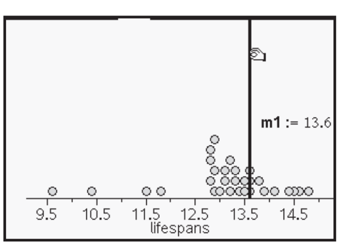
d) If one tusk of very large mass is added, it can increase the mean significantly, even though it is exceptional. This distorts the sense of the average as the measure of the "typical" mass. The median is a more appropriate measure of the typical mass because about half of the masses are less and half are more than the median mass.

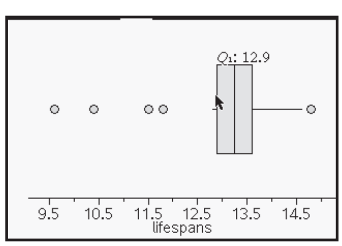
❺. a) In the set of masses for the 40 tusks, the first quartile is Q_1: 8 kg. The second quartile is Q_2: 15 kg. Therefore, $Q_1 \le 12$, $13 \le Q_2$, so masses of 12 and 13 kg fall in the second quartile.

b) Adding the 12-kg and 13-kg tusks to the data set would move the median from 15 kg to 14 kg because the middle numbers in the data set change from 14 and 16 to 14 and 14.

c) Since the data set contains several masses greater than 30 kg, masses of 12-kg and 13-kg are much smaller and probably come from smaller elephants.

❻. a) The display below left shows the dot plot and the vertical line shows that there are 3 cats with life spans of 13.6 years.



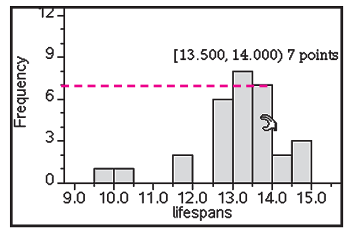


b) The display above right shows the box-and-whisker plot showing that the first quartile is Q_1: 12.9. Placing the pointer on the other parts of the box plot, yields Q_2: 13.25 and Q_3: 13.6. The smallest and larger life spans are 12.8 and 14.6 years respectively.

Exploration 52 *cont'd*

❻. c) The sum of the life spans is 393.9 years, distributed over 30 cats. The mean lifespan is 393.9 ÷ 30 = 13.13 years. That is, the mean is 13.13 years.

d) The display shows the histogram with bin width 0.5 years and alignment 9.0. We see that there are 7 life spans in the interval:
$13.5 \le x \le 14.0$

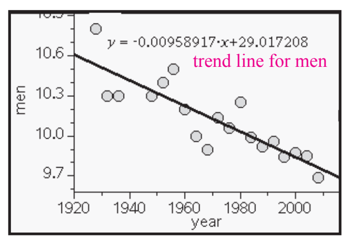


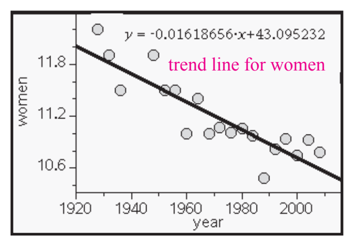
Exploration 53

❶. a) If the points in a scatter plot or *xy*Line plot seem to cluster around a straight line, then a linear regression is the most appropriate line of best fit. However, if the data seem to arch upward or downward, it is reasonable to attempt a quadratic line of best fit.

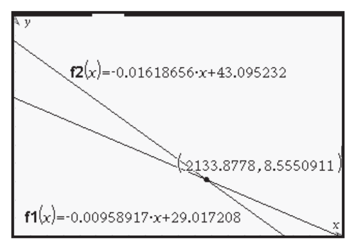
b) (i) linear (ii) quadratic (iii) linear (iv) quadratic

❷ a) The displays below show the trend lines and their equations for the men's and women's winning Olympic times.





b) This display shows the trend lines and their equations for the men's and women's winning Olympic times.
The point of intersection is:
(2133.8777, 8.5550911).



This suggests that the males and females will both have record times of 8.555 seconds in the year 2133. *Example 1* suggested that the winning times in 2133 would be 8.54 seconds. The slight difference in predicted times can be attributed to the effects of rounding. In *Example 1,* the equations of the trend lines were rounded and this made a slight difference in the ultimate prediction.

❸ a) The displays below show the trend lines and their equations for the men's and women's winning Olympic times.