Algebra 1 with TI-nspire

Semester 1

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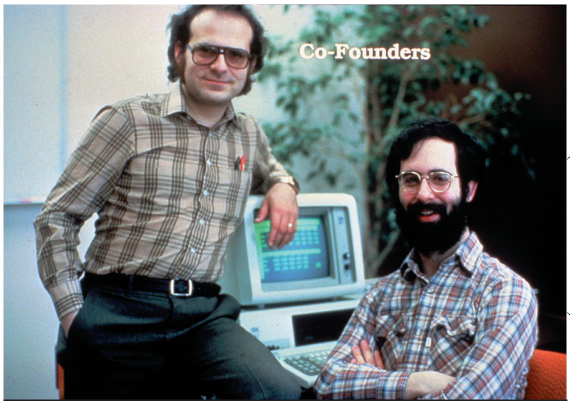
Exploration 18: Sequences & Scatter Plots

In the previous *Explorations,* we saw that the concept of a function evolved from a table of values to its current form introduced by Cantor in 1874. However, the representation of functions continues to evolve. In 1978, Dan Bricklin, while a student at the Harvard Business School, collaborated with Bob Frankston in the development of the first electronic spreadsheet known as *VisiCalc*. This spreadsheet was simply a table of rows and columns, but it had the ability to use data from one or more cells as input to functions defined in other cells. This capability revolutionized accounting practices and brought fame and fortune to its young inventors. Today, *Excel* and *Numbers* are the spreadsheets most commonly used in the business world.



Bob Frankston and Dan Bricklin co-founders of the electronic spreadsheet in 1980

Photo courtesy of Dan Bricklin (www.bricklin.com) and Bob Frankston (www.frankston.com).

The *Examples* in this Exploration show how we can use spreadsheets to represent functions and special functions known as *sequences.* ← A sequence is a function whose domain is the set of integers.

A sequence is an ordered list of numbers such as 1, 3, 5, 7, 9, … , or 2, 4, 8, 16, 32, … . The numbers in a sequence are called the *terms* of the sequence e.g., 5 is the third term in the first sequence and 16 is the fourth term in the second sequence.

The terms of the sequence 1, 3, 5, 7, 9, …, are generated by the formula $2n - 1$, and the terms of the sequence 2, 4, 8, 16, 32, … are generated by the formula 2^n. *Example 1* shows how we generate sequences in TI-*n*spire.

Example 1

a) Display the sequence 1, 2, 3, 4, 5 in column A of a spreadsheet.

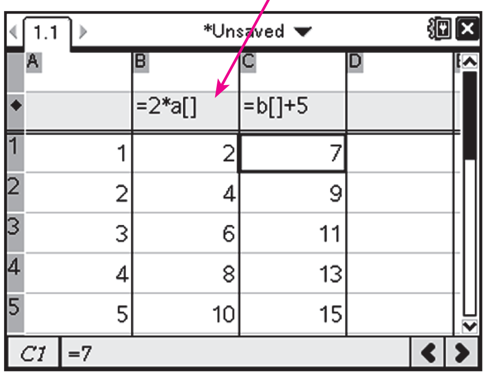
b) Use a formula to display in column B, the numbers that are double those in column A.

c) Use a formula to display in column C, numbers that are 5 more than those in column B.

Solution

a) To access the *Lists & Spreadsheet* application we press: on ▶ ▶ ▶ enter and we enter the numbers from 1 through 5 in column A.

header row of Column B



b) In the header row of column B (see display) we enter:

= 2 × A ctrl (▶ enter

Column B displays the numbers that are double those in column A.

c) In the header row of column C (see display) we enter:

= B ctrl (▶ + 5 enter

Column C displays the numbers that are 5 more than those in column B.

Worked Examples

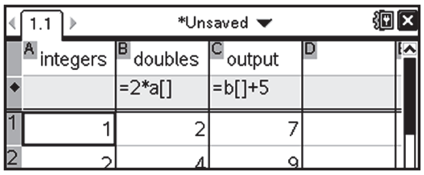
Example 2

a) Construct a scatter plot for the sequence in column C of the spreadsheet in *Example 1* corresponding to the sequence of integers in column A.

b) What do you notice about the shape of the scatter plot? Trace along the points in the scatter plot to display the ordered pairs that define the sequence.

c) Return to the spreadsheet and change the numbers in column A from 1, 2, 3, 4, 5 to 10, 20, 30, 40 50. Then construct the scatter plot as in *part a* above.

Solution



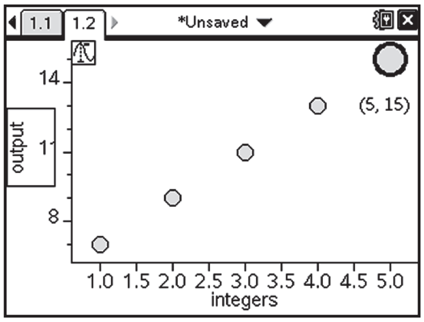
To widen a column press:

menu > **Action** > **Resize** > **Resize Column Width**

Then press ▶ repeatedly and press enter to fix the column width.

a) Before we form a scatter plot, we must name the data in each column of the spreadsheet. To do this, we place the cursor in the top row of the spreadsheet (the row above the header row). In top cell of column A, we enter the name *integers* and press enter. Similarly we name the list of numbers in column B *doubles* and the list of numbers in column C *output* and press enter.

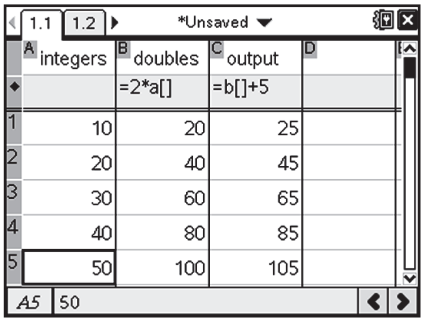
To access the *Data & Statistics* application we press: on ▶ ▶ ▶ ▶ enter.



To display a scatter plot, we move the pointer to the *x*-axis until the prompt *Click to add variable* appears and press enter. We choose *integers* and press enter. Then we move the pointer to the *y*-axis and choose *output.* The scatter plot of the ordered pairs is shown in the display.   .

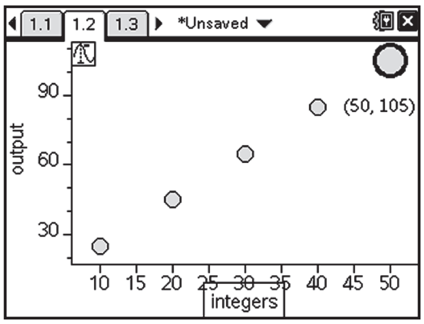
b) We observe that the points in the scatter plot seem to lie on a straight line.

To trace along the graph, we enter: menu > **Analyze** > **Graph Trace**. Pressing ▶ repeatedly moves the cursor through the points on the graph to (5, 15).

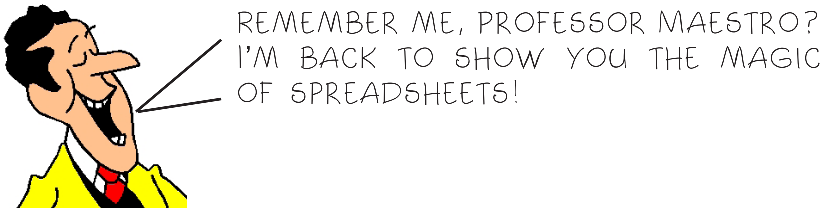


c) To move back one page from the scatter plot in *part b* to the spreadsheet, we press: ctrl ◀. Then, we place the cursor in column A and change the numbers 1, 2, 3, 4, 5 to 10, 20, 30, 40, 50 respectively. As we do this, the numbers in columns B and C change accordingly.

To construct the scatter plot, we proceed as in *part a*) above. We obtain the plot shown in the display, and trace as in *part b* to the point (50, 105).



We see that spreadsheets enable us to have the outputs change automatically when we change inputs!



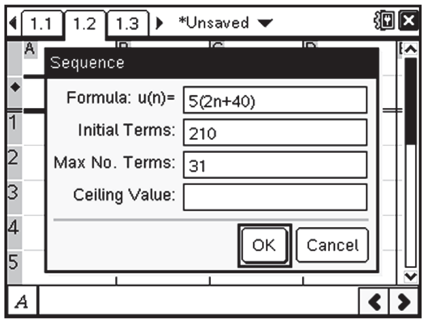
You may recall that in *Exploration 6,* Professor Maestro had students start with the day of their birth that we represent by x. Then the student computed $5(2x + 40)$ and from this number, Professor Maestro would deduce x. His trick was to calculate the "undo" or inverse function. However, in the following *Example 3*, you will discover an easier method using a spreadsheet.

Example 3

Display the values of the function $5(2x + 40)$ on a spreadsheet for integral values of x from 1 to 31. Use the spreadsheet to find the value of x when $5(2x + 40) = 410$.

Solution

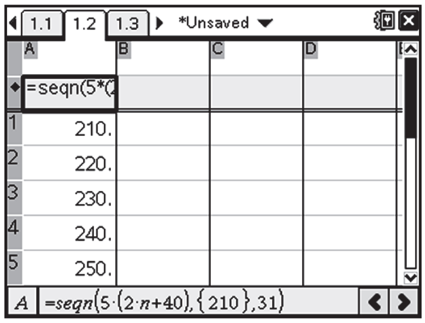
To access the *Lists & Spreadsheets* application we press: [on] ▸ ▸ ▸ [enter].



We place the cursor in the header row of Column A and enter:

[menu] > **Data** > **Generate Sequence**.

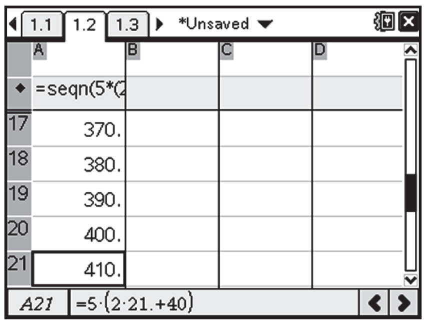
We obtain the *sequence* template shown in the display. Since a sequence is a function defined on the integers, the template uses n rather than x as the variable and $u(n)$ rather than $f(x)$. Therefore, to complete the template, we enter the formula in the box for $u(n)$ as $5(2n + 40)$ as in the screen display.



We press [tab] to move to the next box. The first term of the sequence is its value when $n = 1$, i.e., $5(2 + 40)$ or 210. We enter 210 in the box titled *Initial Terms.*

Since there are only 31 possible days in a month, our sequence has a maximum of 31 terms, so we enter 31 in the box titled *Max No. Terms*. We then tab down to the **OK** box and press [enter].

Column A of our spreadsheet displays the outcomes corresponding to $n = 1, 2, 3, \ldots, 31$.



To find the value x (or in this case n) corresponding to 410, we scroll down column A until we reach 410. The number on the left of 410 is 21 indicating that $x = 21$ is the input number that yields the number 410.

Professor Maestro has, indeed, discovered how to exploit the power of the spreadsheet!

Exercises and Investigations

❶. a) Suppose a friend asks you, “What is a sequence?" Write a sentence or two to describe how you would answer.
b) A quarterback calls out the signals, “4, 6, 3, 2.” Is this a sequence? Explain why or why not.

❷. Identify a pattern. Write the next term of each sequence.
a) 5, 8, 11, 14, __, …
b) –6, –9, –12, –15, __, …
c) 2, 4, 8, 16, __, …
d) 1, 4, 9, 16, __, …

❸. Give an example of a function that is *not* a sequence.

❹. The *n*th term of an infinite sequence is defined by $u(n) = 1/n$.
Note: We use $u(n)$ instead of $f(n)$ for sequences.
a) Write the initial term (i.e., first term) of the sequence.
b) Write the 20th term of the sequence as a decimal fraction.
c) Write the 100th term of the sequence as a decimal fraction.

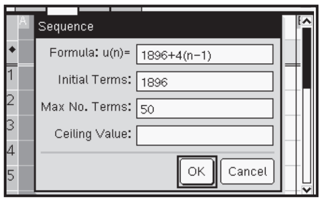
❺. The sequence of even integers is 0, 2, 4, 6, … .
a) Write the initial term of this sequence.
b) Write the 9th term of this sequence.
c) Write an expression for the *n*th term of this sequence.
d) Use your expression to calculate the 50th term.

❻. Write an expression for the *n*th term $u(n)$ of each of the sequences in *Exercise* ❷.

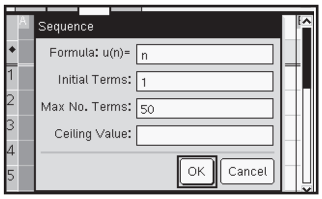
❼. The Olympic Games originated in ancient Greece around 776 B.C. In 1896, the Modern Olympic Games were instituted. It was agreed that the first (summer) games would be held in 1896 and every four years thereafter – known as the *Olympic Years*. The photo shows the Olympic facility where the athletes trained in preparation for the games.


photo courtesy of Teresa Kelly

a) Write the first four terms of the sequence of Olympic Years.
b) In what year were the 9th Olympic Games held?
c) Write an expression for the *n*th Olympic Year.
d) To open a spreadsheet in TI-*n*spire press [on] ▶ ▶ ▶ [enter].
Then enter [menu] > **Data** > **Generate Sequence** to obtain the sequence template.
Complete the template as in the display and press [enter].
e) Scroll down to the 9th row of Column A to verify your answer in part b.
f) Scroll to find the number of the Olympic games that were held in 1996.



❽. Complete *Exercise* ❼ before proceeding. Then follow the steps in that exercise to obtain the sequence template.
a) To generate the sequence of natural numbers from 1 to 50, complete the template as in the display and click OK.
b) Then in the top row of Column A enter *natnos* to name that sequence.
c) Follow the procedure in *Example 2* to create a scatter plot of *olympicyr* vs. *natnos.* Describe the shape of the plot.



d) Trace along the scatter plot to check your answer to ❼f.

❾. Complete *Exercise* ❽ before proceeding. Then open a spreadsheet by pressing [on] ▶ ▶ ▶ [enter].
- In the top row of Column A, enter *natnos.*
- In the header row of Column B, enter =a[ ] + 1.
- In the header row of Column C, enter =a[ ]*b[ ].
- In the header row of Column D, enter =c[ ] /2.

a) How are the numbers in Columns A and B related?
b) How are the numbers in Column C related to the numbers in columns A and B ?
c) How are the numbers in Columns C and D related?
d) Suppose a number in column A is represented by *n*. Write expressions in *n* for the numbers in Columns B, C, and D.
e) Use your expression in *n* to calculate the number in row 50 of column D. Compare this with the number you found in the TI-*n*spire Investigation in *Exploration 13*.

❿. Professor Maestro is at it again! These are his instructions.

A	B	C	D	E
Start with a positive integer *n*.	Write *n* + 1 in box B	Write *n* – 1 in box C	Write in box D the product of box B and box C	Add 1 to Box D. Give the result.

Open a spreadsheet in TI-*n*spire by pressing [on] ▶ ▶ ▶ [enter].
- In the top row of Column A, enter *natnos.*
- In the header row of Column B, enter: =a[ ] + 1.
- Enter the appropriate expressions in the header rows of Columns C, D and E. Examine the numbers in Column E and explain how Professor Maestro determines the input number *n* from the output number in Column E.

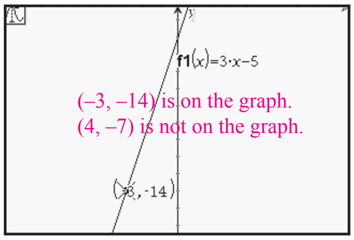
TI-nspire Investigation

What is the 50th Triangular Number?

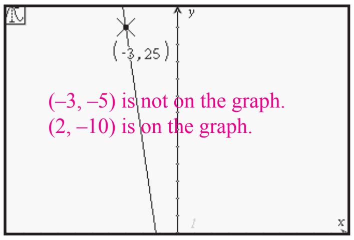
Follow the instructions in *Exercise* ❾ to create a spreadsheet with the natural numbers (*natnos*) from 1 to 50 in Column A and the triangular numbers in Column D. Then use the procedure in *Example 2a* to construct a scatter plot of *triangularnos* vs. *natnos*. Trace along your scatter plot to display the 50th triangular number.

Exploration 17 *cont'd*

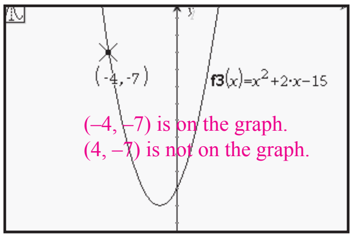
❽. a)



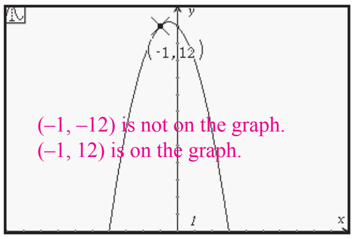
b)



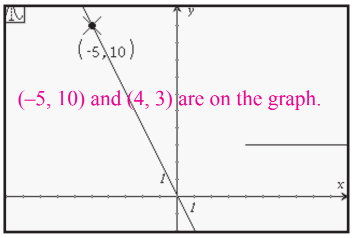
c)



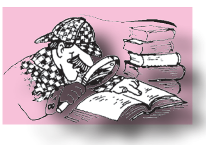
d)



e)



Hint for the TI-nspire Investigation



The number of dots in an interval on the horizontal axis indicate the number of people with IQ in that range. Most people cluster close to an IQ between 90 and 110.

Exploration 18

❶. a) A sequence is an ordered list that assigns a number to each of the natural numbers 1, 2, 3,… in turn.
b) Yes, the signals constitute a finite sequence of 4 terms. The first term is 4, the second term is 6, the third term is 3, and the last term is 2.

❷. a) Each term is 3 more than the previous term. The next term is 17.
b) Each term is 3 less than the previous term. The next term is –18.
c) Each term is double the previous term. The next term is 32.
d) The nth term is n^2. The next term is 5^2 or 25.

❸. Answers will vary, but a function whose domain is not the natural numbers is not a sequence. e.g., $f(x) = x^2$ is not a sequence if the domain of $f(x)$ is the set of real numbers. Also a set of ordered pairs such as: {(2,3), (5, 9), (1, 7)} is a function but not a sequence because this is not an ordered list.

❹. a) initial term is 1 b) 20th term is 1/20 or 0.05
c) 100th term is 1/100 or 0.01

❺. a) initial term is 0 b) 9th term is 2 × 8 or 16
c) nth term is $2(n – 1)$ d) 50th term is 2(50 – 1) or 98

❻. a) nth term is $3n + 2$ b) nth term is $–3n – 3$
c) nth term is 2^n d) nth term is n^2

Exploration 18 *cont'd*

❼. a) 1896, 1900, 1904, 1908 b) 1928
c) nth Olympic year is $1896 + 4(n – 1)$
e) The display shows 1928 f) The display shows the 26*th*.

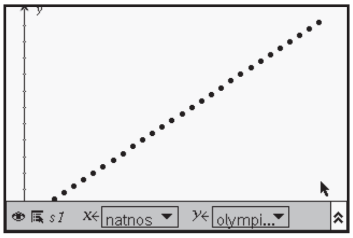
	A olympi...	B	C	D
◆	=seqn(189			
5	1912			
6	1916			
7	1920			
8	1924			
9	1928			

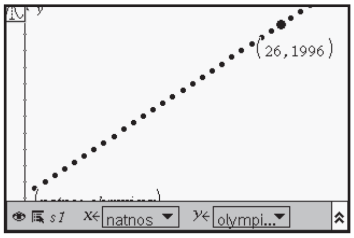
A9 =1896+4·(9−1)

	A olympi...	B	C	D
◆	=seqn(189			
22	1980			
23	1984			
24	1988			
25	1992			
26	1996			

A26 =1896+4·(26−1)

❽. c) The plot is a straight line d) The trace yields (26, 1996).





❾. a) The numbers in Column B are greater by 1 than those in A.
b) The numbers in Column C are the product of those in A and B.
c) The numbers in Column D are half of those in C.
d) The expressions in Columns B, C and D are respectively: $n + 1$, $n(n + 1)$, and $n(n + 1)/2$.
e) The number in row 50 of column D is 50 × 51 ÷ 2 or 1275.

	A natnos	B	C	D triangul...
◆	=seqn(n,{1	=a[]+1	=a[]*b[]	=c[]/2
46	46	47	2162	1081
47	47	48	2256	1128
48	48	49	2352	1176
49	49	50	2450	1225
50	50	51	2550	1275

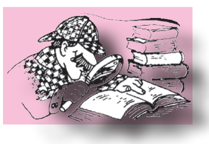
D50 =1275

❿. If n denotes the number in Column A, then the expressions defining Columns B, C, D, and E are respectively, $n + 1$, $n – 1$, $(n + 1) × (n–1)$, and n^2. To determine the number in Column A from a given number in Column E, Professor Maestro merely takes the square root of the number in Box E.

	B	C	D	E
◆	=a[]+1	=a[]−1	=b[]*c[]	=d[]+1
1	2.	0.	0.	1.
2	3	1	3	4
3	4	2	8	9
4	5	3	15	16
5	6	4	24	25

E5 =25

Hint for the TI-nspire Investigation



The scatter plot is shown in the display with the 50th triangular number highlighted in the trace.